SB-3539/200

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M. A. / M. Sc. (Part II) Examination

March / April – 2011

Mathematics : Paper-5008

(Advanced Special Functions)

(New Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. A. / M. Sc. (Part II)**

Name of the Subject :
☛ **Mathematics : Paper-5008**

☛ Subject Code No. : **3 5 3 9** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.
(3) Figure to the right indicates marks of the question.
(4) Notations and conventions are all standard.

1 (a) If $P = q+1$ and no a_m is zero or a nagative integer **8**

show that $\frac{1}{2\lambda i}\int_B \frac{\Gamma(-s)\,(-\tau)^s \prod_{n=1}^{P}\Gamma(a_m+s)}{\prod_{j=1}^{q}\Gamma(b_j+s)}\,ds$, where B is Barnes contour, is an analytic function of z in the cut plane $|\arg(-\tau)| < \pi$.

(b) Define contiguous function ${}_pF_q$. Obtain the relation **6**

$$[(1-x)\alpha_1 + (A-B)x]F = (1-x)\alpha_1 F(\alpha_1+) - \sum_{j=1}^{q} U_j F(B_j+),\ P = q+1$$

OR

1 (a) With usual notation prove that **8**

$$ {}_3F_2\left[\begin{matrix} a, & b, & c; \\ 1+a-b & & 1+a-c; \end{matrix} \; 1\right] = \frac{\Gamma\left(1+\frac{a}{2}\right)\Gamma(1+a-b)\,\Gamma(1+a-c)\,\Gamma\left(1+\frac{a}{2}-b-c\right)}{\Gamma(1+a)\,\Gamma\left(1+\frac{a}{2}-b\right)\Gamma\left(1+\frac{a}{2}-c\right)\Gamma(1+a-b-c)} $$

(b) Derive the Generalized hypergeometric differential equation $\left[\varphi\prod_{j=1}^{q}\left(\varphi+b_j-1\right)-z\prod_{i=1}^{p}\left(\varphi+a_i\right)\right]W=0$ **6**

2 (a) If neither $(a-b)$ nor $(a-c)$ nor a is a negative integer, then prove that **8**

$$ {}_3F_2\left[\begin{matrix} a, & b, & c, \\ 1+a-b, & & 1+a-c, \end{matrix} \; x\right] = (1-x)^{-a}\, {}_3F_2\left[\begin{matrix} \frac{a}{2}, & \frac{a}{2}+\frac{1}{2} & 1+a-b-c, \\ 1+a-b, & & 1+a-c, \end{matrix} \; \frac{-4x}{(1-x)^2}\right] $$

(b) If $k(k)$ is the complete elliptic integral of the second kind, then prove that **6**

$$ \int_0^t k\left(\sqrt{x(t-x)}\right)dx=\pi\sin^{-1}\left(\frac{t}{2}\right) $$

OR

2 (a) If n is a non-negative integer and if a,b,c are independent of n, prove that **8**

$$ {}_3F_2\left[\begin{matrix} -n, \; a, \; b, \\ c, \; 1-c+a+b-n, \end{matrix} \; 1\right] = \frac{(c-a)_n\,(c-b)_n}{(c)_n\,(c-a-b)_n} $$

(b) Show that **6**

$$ \int_0^t x^{1/2}(t-x)^{-1/2}\left[1-x^2(t-x)^2\right]^{-1/2}dx $$

$$ =\frac{\pi}{2}t\; {}_2F_1\left[\begin{matrix} \frac{1}{4}, \; \frac{3}{4}; \\ 1; \end{matrix} \; \frac{t^4}{16}\right] $$

3 (a) State and prove the kummer's first formula for the confluent hypergeometric function ${}_1F_1(a; b; z)$. **8**

(b) Define the orthogonality of a simple set of real polynomials $\phi_n(x)$. Show that the set $\phi_n(x)$ is orthogonal if and only if $\int_a^b \omega(x) x^k \phi_n(x) dx = 0$ for $k = 0, 1, \ldots, n-1$ and $a < x < b$. **6**

OR

3 (a) The poynomial $f_n(x)$ is defined by **8**

$$(1-t)^{-c} \psi\left(\frac{-4xt}{(1-t)^2}\right) = \sum_{n=0}^{\infty} f_{n(x)} t^n \text{, where}$$

$\psi(u) = \sum_{n=0}^{\infty} \gamma_n u^n$, $\gamma_0 \neq 0$, then prove that

$$x^n = \frac{(c)_{2n}}{2^{2n} \gamma_n} \sum_{k=0}^{n} \frac{(-1)^k \ (c+2k) f_k(x)}{(n-k)! \ (c)_{n+k+1}}$$

(b) Let $e^t \psi(xt) = \sum_{n=0}^{\infty} b_n(x) t^n$, $\psi(u) = \sum_{n=0}^{\infty} \gamma_n u^n$ then for arbitrary c prove that $(1-t)^{-c} F\left(\frac{xt}{1-t}\right) = \sum_{n=0}^{\infty} (c)_n b_n(x) t^n$ in which $F(u) = \sum_{n=0}^{\infty} (c)_n \gamma_n u^n$. **6**

4 (a) With usual notation prove that **6**

$$L_n^{(\alpha)}(x) = \frac{(1+\alpha)_n}{(c)_n} \sum_{k=0}^{n} \frac{(1+\alpha-c)_k \, L_{n-k}^{(2c-\alpha-2)}(x) L_k^{(\alpha)}(x)}{(1+\alpha)_k}$$

(b) Obtain the relation **4**

$$L_n^{(\alpha)}(x) = L_{n-1}^{(\alpha)}(x) + L_n^{(\alpha-1)}(x)$$

(c) Show that **4**

$$L_n^{(\alpha)}(xy) = \sum_{k=0}^{\infty} \frac{(1+\alpha)_n (1-y)^{n-k} y^k L_k^{(\alpha)}(x)}{(n-k)!\,(1+\alpha)_k}.$$

OR

4 (a) Prove that $\int_0^{\infty} e^{-x} x^{\alpha} \left\{ L_n^{(\alpha)}(x) \right\}^2 dx = \frac{(1+\alpha+n)}{n!}$ **6**

(b) Prove that $L_n^{(\alpha)}(x) = \frac{x^{-\alpha} e^{x}}{n!} D^n \left(e^{-x} x^{n+\alpha} \right)$ **4**

(c) Obtain the polynomials $L_0^{(\alpha)}(x), L_1^{(\alpha)}(x), L_2^{(\alpha)}(x), L_3^{(\alpha)}(x)$. **4**

5 (a) With usual notation prove that **8**

$$P_n^{(\alpha,\beta)}(x) = (1-t)^{-(1+\alpha+\beta)}\ {}_2F_1\left[\begin{matrix} 1+\alpha+\beta, & 2+\alpha+\beta; & \frac{2t(x-1)}{(1-t)^2} \\ & 1+\alpha; & \end{matrix}\right]$$

(b) Prove that **6**

$$\sum_{n=0}^{\alpha} \frac{P_n^{(\alpha,\beta)}(x)\, t^n}{(1+\alpha)_n\, (1+\beta)_n} = {}_0F_1\left[\begin{matrix} -; & \frac{t(x-1)}{2} \\ 1+\alpha; & \end{matrix}\right] {}_0F_1\left[\begin{matrix} -; & \frac{t(x+1)}{2} \\ 1+\beta; & \end{matrix}\right]$$

OR

5 (a) With usual notation prove that **8**

$$F_4\left(a, b; c, 1-c+a+b; \frac{-x}{(1-x)(1-y)}, \frac{-y}{(1-x)(1-y)} \right)$$

$$= {}_2F_1\left[\begin{matrix} a, & b; & \frac{-x}{1-x} \\ & c; & \end{matrix}\right] {}_2F_1\left[\begin{matrix} a, & b; & \frac{-y}{1-y} \\ & 1-c+a+b; & \end{matrix}\right]$$

(b) With usual notation prove that **6**

$$\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x) t^n = 2^{\alpha+\beta} \rho^{-1} (1+t+\rho)^{-B} (1-t+\rho)^{-\alpha}$$

where $\rho^2 = 1 - 2xt + t^2$.
